Communication Systems

Amplitude Modulation :

DSB-SC :

$$u(t) = A_C\, m(t) \cos 2\pi f_c t$$

Power $P = \frac{A_C^2}{2} P_M$

Conventioanal AM :

$u(t) = A_C[1 + m(t)] \, \text{Cos}\, 2\pi f_c t$. as long as $|m(t)| \le 1$ demodulation is simple .

Practically $m(t) = a\, m_n(t)$.

Modulation index $a = \frac{m(t)}{m_n(t)}$, $m_n(t) = \frac{m(t)}{\max|m(t)|}$

Power $= \frac{A_C^2}{2} + \frac{A_C^2\, a^2}{4}$

SSB-AM :

→ Square law Detector $SNR = \frac{2}{K_a\, m(t)}$

Square law modulator

↓

$K_a = 2a_2 / a_1$ → amplitude Sensitivity

Envelope Detector $R_s C$ (i/p) << $1/f_c$ $R_l C$ (o/P) >> $1/f_c$ $R_l C << 1/\omega$

$$\frac{1}{R_l C} \ge \frac{\omega_m\, \mu}{\sqrt{1-\mu^2}}$$

Frequency & Phase Modulation : Angle Modulation :-

$$u(t) = A_C \, \text{Cos}\, (2\pi f_c t + \emptyset(t))$$

$$\emptyset(t) \begin{cases} K_p\, m(t) & \rightarrow PM \\ 2\pi K_f \int_{-\infty}^{t} m(t) . dt & \rightarrow FM \end{cases}$$

K_p & K_f phase & frequency deviation constant

→ max phase deviation $\Delta\emptyset = K_p \max|m(t)|$

→ max frequency deviation $\Delta f = K_f \max|m(t)|$

Bandwidth :

Effective Bandwidth $B_C = 2(\beta + 1) f_m$ → 98% power

Noise in Analog Modulation :-

→ $(SNR)_{\text{Base Band}} = \left(\frac{S}{N}\right)_0 = \frac{P_m}{P_n} = \frac{P_R}{N_0 B}$

$R = m(t) \cos 2\pi f_c$ ∴ $P_R = P_m / 2$

$\rightarrow (SNR)_{DSB-SC} = \frac{P_m/4}{P_{nI/4}} = \frac{P_m}{2N_0B} = \frac{2P_R}{2N_0B} = \frac{P_R}{N_0R} = \left(\frac{S}{N}\right)_0 = (SNR)_{Base\ band}$

$\rightarrow (SNR)_{SSB-SC} = \frac{P_m/4}{P_{ni/4}} = \frac{P_m}{N_0B} = \frac{P_R}{N_0B} = \left(\frac{S}{N}\right)_0 = (SNR)_{Base\ band}.$

$\left(\frac{S}{N}\right)_{com\ AM} = \frac{\mu^2 P_m}{1+\mu^2 P_m} \cdot \frac{P_R}{N_0B} = \eta \left(\frac{S}{N}\right)_{Base\ Band}$ $\eta = \frac{\mu^2 P_m}{1+\mu^2 P_m}$

Noise in Angle Modulation :-

$$\left(\frac{S}{N}\right)_0 = \begin{cases} \beta_p^2 P_{M_n} \left(\frac{S}{N}\right)_b, & PM \\ 3\beta_f^2 P_{M_n} \left(\frac{S}{N}\right)_b, & FM \end{cases}$$

PCM :-

→ Min. no of samples required for reconstruction = $2\omega = f_s$; ω = Bandwidth of msg signal .

→ Total bits required = $v f_s$ bps . $v \rightarrow$ bits / sample

→ Bandwidth = $R_b/2 = v f_s / 2 = v \cdot \omega$

→ SNR = 1.76 + 6.02 v

→ As Number of bits increased SNR increased by 6 dB/bit . Band width also increases.

Delta Modulation :-

→ By increasing step size slope over load distortion eliminated [Signal raised sharply]

→ By Reducing step size Grannualar distortion eliminated . [Signal varies slowly]

Digital Communication

Matched filter:

→ impulse response $a(t) = P^*(T-t)$. $P(t) \rightarrow$ i/p

→ Matched filter o/p will be max at multiples of 'T' . So, sampling @ multiples of 'T' will give max SNR (2nd point)

→ matched filter is always causal $a(t) = 0$ for $t < 0$

→ Spectrum of o/p signal of matched filter with the matched signal as i/p ie, except for a delay factor ; proportional to energy spectral density of i/p.

$$\emptyset_0(f) = H_{opt}(f)\,\emptyset(f) = \emptyset(f)\,\emptyset^*(f)\,e^{-2\pi fT}$$

$$\emptyset_0(f) = |\emptyset(f)|^2\,e^{-j2\pi fT}$$

→ o/p signal of matched filter is proportional to shifted version of auto correlation fine of i/p signal

$\emptyset_0(t) = R_\emptyset(t-T)$

At t = T $\emptyset_0(T) = R_\emptyset(0)$ → which proves 2^{nd} point

Cauchy-Schwartz in equality :-

$$\int_{-\infty}^{\infty} |g_1^*(t)\, g_2(t)\, dt|^2 \le \int_{-\infty}^{\infty} g_1^2(t)\, dt \int_{-\infty}^{\infty} |g_2(t)|^2\, dt$$

If $g_1(t) = c\, g_2(t)$ then equality holds otherwise '<' holds

Raised Cosine pulses :

$$P(t) = \frac{\mathrm{Sin}(\frac{\pi t}{T})}{(\frac{\pi t}{T})} \cdot \frac{\mathrm{Cos}\,(\frac{\pi \alpha t}{T})}{1-4\alpha^2 t\, T^2}$$

$$P(f) = \begin{cases} T, & |f| \le \frac{1-\alpha}{2T} \\ T\ \cos^2\left(\frac{\pi t}{2\alpha}\left(|f| - \frac{1-\alpha}{2T}\right)\right); & \frac{1-\alpha}{2T} \le |f| \le \frac{1+\alpha}{2T} \\ 0, & |f| > \frac{1+\alpha}{2T} \end{cases}$$

- Bamdwidth of Raised cosine filter $f_B = \frac{1+\alpha}{2T}$ ⇒ Bit rate $\frac{1}{T} = \frac{2f_B}{1+\alpha}$
  α → roll of factor
  T → signal time period

→ For Binary PSK $P_e = Q\left(\frac{d}{2\sigma}\right) = Q\left(\sqrt{\frac{2\varepsilon_s}{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{\varepsilon_s}{N_0}}\right)$.

→ 4 PSK $P_e = 2Q\left(\sqrt{\frac{2\varepsilon_b}{N_0}}\right)\left[1 - \frac{1}{2}\,Q\left(\sqrt{\frac{2\varepsilon_b}{N_0}}\right)\right]$

FSK:-
For BPSK

$$P_e = Q\left(\frac{d}{2\sigma}\right) = Q\left(\sqrt{\frac{\varepsilon_s}{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{\varepsilon_s}{2N_0}}\right)$$

→ All signals have same energy (Const energy modulation)

→ Energy & min distance both can be kept constant while increasing no. of points . But Bandwidth Compramised.
→ PPM is called as Dual of FSK .
→ For DPSK $P_e = \frac{1}{2}\, e^{-\varepsilon_b/N_0}$

→ Orthogonal signals require factor of '2' more energy to achieve same P_e as anti podal signals

→ Orthogonal signals are 3 dB poorer than antipodal signals. The 3dB difference is due to distance b/w 2 points.

→ For non coherent FSK $P_e = \frac{1}{2} e^{-\varepsilon_b/N_0}$

→ FPSK & 4 QAM both have comparable performance .

→ 32 QAM has 7 dB advantage over 32 PSK.

- Bandwidth of Mary PSK = $\frac{2}{T_s} = \frac{2}{T_{b\log_2^m}}$; $S = \frac{\log_2^m}{2}$
- Bandwidth of Mary FSK = $\frac{M}{2T_s} = \frac{M}{2T_b\log_2^m}$; $S = \frac{\log_2^m}{m}$
- Bandwidth efficiency $S = \frac{R_b}{B.W}$.
- Symbol time $T_s = T_b \log_2^m$
- Band rate $= \frac{\text{Bit rate}}{\log_2^m}$

Signals & Systems

→ Energy of a signal $\int_{-\infty}^{\infty} |x(t)|^2 \, dt = \sum_{n=-\infty}^{\infty} |x[n]|^2$

→ Power of a signal $P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$

→ $x_1(t) \to P_1$; $x_2(t) \to P_2$
$x_1(t) + x_2(t) \to P_1 + P_2$ iff $x_1(t)$ & $x_2(t)$ orthogonal

→ Shifting & Time scaling won't effect power . Frequency content doesn't effect power.

→ if power = ∞ → neither energy nor power signal
Power = 0 ⇒ Energy signal
Power = K ⇒ power signal

→ Energy of power signal = ∞ ; Power of energy signal = 0

→ Generally Periodic & random signals → Power signals
Aperiodic & deterministic → Energy signals

Precedence rule for scaling & Shifting :

x(at + b) → (1) shift x(t) by 'b' → x(t + b)
(2) Scale x(t + b) by 'a' → x(at + b)

x(a (t + b/a)) → (1) scale x(t) by a → x(at)
(2) shift x(at) by b/a → x (a (t+b/a)).

→ $x(at+b) = y(t) \Rightarrow x(t) = y\left(\frac{t-b}{a}\right)$

- Step response $s(t) = h(t) * u(t) = \int_{-\infty}^{t} h(t)dt$ $S'(t) = h(t)$
  $S[n] = \sum_{k=0}^{n} h[n]$ $h[n] = s[n] - s[n-1]$
- $e^{-at} u(t) * e^{-bt} u(t) = \frac{1}{b-a} [e^{-at} - e^{-bt}] u(t)$.
- $A_1 \text{Rect}(t / 2T_1) * A_2 \text{Rect}(t / 2T_2) = 2 A_1 A_2 \min(T_1, T_2)$ trapezoid (T_1, T_2)
- Rect (t / 2T) * Rect (t / 2T) = 2T tri(t / T)

Hilbert Transform Pairs :

$\int_{-\infty}^{\infty} e^{-x^2/2\sigma^2} dx = \sigma\sqrt{2\pi}$; $\int_{-\infty}^{\infty} x^2 e^{-x^2/2\sigma^2} dx = \sigma^3 \sqrt{2\pi}$ $\sigma > 0$

Laplace Transform :-

$x(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)\, e^{st}\, ds$

$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, ds$

Initial & Final value Theorems :

x(t) = 0 for t < 0 ; x(t) doesn't contain any impulses /higher order singularities @ t =0 then

$x(0^{+}) = \lim_{s\to\infty} s\, X(s)$

$x(\infty) = \lim_{s\to 0} s\, X(s)$

Properties of ROC :-

1. X(s) ROC has strips parallel to jω axis

2. For rational laplace transform ROC has no poles

3. x(t) → finite duration & absolutely integrable then ROC entire s-plane

4. x(t) → Right sided then ROC right side of right most pole excluding pole s = ∞

5. x(t) → left sided ROC left side of left most pole excluding s= - ∞

6. x(t) → two sided ROC is a strip

7. if x(t) causal ROC is right side of right most pole including s = ∞

8. if x(t) stable ROC includes jω-axis

Z-transform :-

$x[n] = \frac{1}{2\pi j}\oint x(z)\, z^{n-1}\, dz$

$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$

Initial Value theorem :

If x[n] = 0 for n < 0 then $x[0] = \lim_{z\to\infty} X(z)$

Final Value theorem :-

$\lim_{\to\infty} x[n] = \lim_{z\to 1}(z-1)\, X(z)$

Properties of ROC :-

1.ROC is a ring or disc centered @ origin
2. DTFT of x[n] converter if and only if ROC includes unit circle
3. ROC cannot contain any poles

4. if x[n] is of finite duration then ROC is enter Z-plane except possibly 0 or ∞
5. if x[n] right sided then ROC → outside of outermost pole excluding z = 0
6. if x[n] left sided then ROC → inside of innermost pole including z =0
7. if x[n] & sided then ROC is ring
8. ROC must be connected region
9.For causal LTI system ROC is outside of outer most pole including ∞
10.For Anti Causal system ROC is inside of inner most pole including '0'
11. System said to be stable if ROC includes unit circle .
12. Stable & Causal if all poles inside unit circle
13. Stable & Anti causal if all poles outside unit circle.

Phase Delay & Group Delay :-
When a modulated signal is fixed through a communication channel , there are two different delays to be considered.

(i) Phase delay:
Signal fixed @ o/p lags the fixed signal by $\emptyset(\omega_c)$ phase

$\tau_P = -\frac{\emptyset(\omega_c)}{\omega_c}$ where $\emptyset(\omega_c) =$ K H(jω)

↓

Frequency response of channel

Group delay $\tau_g = -\frac{d\emptyset(\omega)}{d\omega}\Big|_{\omega=\omega_c}$ for narrow Band signal

↓

Signal delay / Envelope delay

Probability & Random Process:-

→ $P(A/B) = \frac{P(A\cap B)}{P(B)}$

→ Two events A & B said to be mutually exclusive /Disjoint if $P(A \cap B) = 0$

→ Two events A & B said to be independent if $P(A/B) = P(A) \Rightarrow P(A \cap B) = P(A)\,P(B)$

→ $P(Ai / B) = \frac{P(Ai \cap B)}{P(B)} = \frac{P\left(\frac{B}{Ai}\right) P(Ai)}{\sum_{i=1}^{n} P\left(\frac{B}{Ai}\right) P(Ai)}$

CDF :-
Cumulative Distribution function $F_x(x) = P\{X \le x\}$

Properties of CDF :

- $F_x(\infty) = P\{X \le \infty\} = 1$
- $F_x(-\infty) = 0$
- $F_x(x_1 \le X \le x_2) = F_x(x_2) - F_x(x_1)$
- Its Non decreasing function
- $P\{X > x\} = 1 - P\{X \le x\} = 1 - F_x(x)$

PDF :-

Pdf = $f_x(x) = \frac{d}{dx} F_x(x)$

Pmf = $f_x(x) = \sum_{i=-\infty}^{\infty} P\{X = x_i\}\ \delta(x = x_i)$

Properties:-

- $f_x(x) \geq 0$
- $F_x(x) = f_x(x) * u(x) = \int_{-\infty}^{x} f_x(x)\,dx$
- $F_x(\infty) = \int_{-\infty}^{\infty} f_x(x)\,dx = 1$ so, area under PDF = 1
- $P\{x_1 < X \leq x_2\} = \int_{x_1}^{x_2} f_x(x)dx$

Mean & Variance :-

Mean $\mu_x = E\{x\} = \int_{-\infty}^{\infty} x\, f_x(x)\,dx$

Variance $\sigma^2 = E\{(X - \mu_x)^2\} = E\{x^2\} - \mu_x^2$

$\rightarrow E\{g(x)\} = \int_{-\infty}^{\infty} g(x)\, f_x(x)\,dx$

Uniform Random Variables :

Random variable X ~ u(a, b) if its pdf of form as shown below

$$f_x(x) = \begin{cases} \frac{1}{b-a} ; a < x \leq b \\ 0 , else \end{cases}$$

$$F_x(x) = \begin{cases} 1 ; x > b \\ \frac{x-a}{b-a} ; a < x < b \\ 0 ; else \end{cases}$$

Mean = $\frac{a+b}{2}$

Variance = $(b - a)^2 / 12$ $\quad E\{x^2\} = \frac{a^2 + ab + b^2}{3}$

Gaussian Random Variable :-

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\ e^{-(x-\mu)^2/2\sigma^2}$$

$X \sim N(\mu_1 \sigma^2)$

Mean = $\int_{-\infty}^{\infty} x\ \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}\,dx = \mu$

Variance = $\frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} x^2 \; e^{-(x-\mu)^2/2\sigma^2} \, dx = \sigma^2$

Exponential Distribution :-

$f_x\,(x) = \lambda\, e^{-\lambda x}\, u(x)$

$F_x(x) = (\,1 - e^{-\lambda x}\,)\, u(x)$

Laplacian Distribution :-

$f_x(x) = \frac{\lambda}{2}\, e^{-\lambda\,|x|}$

Multiple Random Variables :-

- $F_{XY}\,(x\,,\,y) = P\,\{\,X \le x\,,\,Y \le y\,\}$
- $F_{XY}\,(x\,,\,\infty) = P\,\{\,X \le x\,\} = F_x\,(x)$; $F_{xy}\,(\infty\,,\,y) = P\,\{\,Y < y\,\} = F_Y(y)$
- $F_{xy}\,(-\infty,\,y) = F_{xy}\,(x,\,-\infty) = F_{xy}\,(-\infty,\,-\infty) = 0$
- $f_x(x) = \int_{-\infty}^{\infty} f_{xy}\,(x, y)\,dy$; $f_Y\,(y) = \int_{-\infty}^{\infty} f_{xy}\,(x, y)\,dx$
- $F_{Y/X}\left(\frac{Y}{X} \le x\right) = \frac{P\{Y \le y,\;\, X \le x\}}{P\{X \le x\}} = \frac{F_{XY}(x,y)}{F_X(x)}$
- $f_{Y/X}(y/x) = \frac{f_{xy}(x,y)}{f_x(x)}$

Independence :-

- X & Y are said to be independent if $F_{XY}(x\,,\,y) = F_X(x)\,F_Y(y)$

$\Rightarrow f_{XY}\,(x, y) = f_X(x)\,.\,f_X(y)$ $\quad$ $P\,\{\,X \le x,\,Y \le y\} = P\,\{\,X \le x\}\,.\,P\{Y \le y\}$

Correlation:

$\text{Corr}\{\,XY\} = E\,\{XY\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{xy}(x, y).\,xy.\,dx\,dy$

If E { XY} = 0 then X & Y are orthogonal .

Uncorrelated :-

Covariance = $\text{Cov}\,\{XY\} = E\,\{\,(X - \mu_x)\,(Y - \mu_y\}$

$= E\,\{xy\} - E\,\{x\}\,E\{y\}.$

If covariance = 0 $\Rightarrow E\{xy\} = E\{x\}\,E\{y\}$

- Independence → uncorrelated but converse is not true.

Random Process:-

Take 2 random process X(t) & Y(t) and sampled @ t_1, t_2

$X(t_1)\,,\,X(t_2)\,,\,Y(t_1)\,,\,Y\,(t_2)$ → random variables

→ Auto correlation $R_x(t_1, t_2) = E\,\{X(t_1)\,X(t_2)\,\}$

→ Auto covariance $C_x(t_1, t_2) = E\{ X(t_1) - \mu_x(t_1))(X(t_2) - \mu_x(t_2)\} = R_x(t_1, t_2) - \mu_x(t_1)\, \mu_x(t_2)$
→ cross correlation $R_{xy}(t_1, t_2) = E\{ X(t_1)\, Y(t_2)\}$
→ cross covariance $C_{xy}(t_1, t_2) = E\{ X(t_1) - \mu_x(t_1))(Y(t_2) - \mu_y(t_2)\} = R_{xy}(t_1, t_2) - \mu_x(t_1)\, \mu_y(t_2)$

→ $C_{XY}(t_1, t_2) = 0 \Rightarrow R_{xy}(t_1, t_2) = \mu_x(t_1)\, \mu_y(t_2)$ → Un correlated
→ $R_{XY}(t_1, t_2) = 0 \Rightarrow$ Orthogonal cross correlation = 0
→ $F_{XY}(x, y \,!\, t_1, t_2) = F_x(x! \, t_1)\, F_y(y \,!\, t_2)$ → independent

Properties of Auto correlation :-

- $R_x(0) = E\{ x^2 \}$
- $R_x(\tau) = R_x(-\tau)$ → even
- $|R_x(\tau)| \leq R_x(0)$

Cross Correlation

- $R_{xy}(\tau) = R_{yx}(-\tau)$
- $R_{xy}^2(\tau) \leq R_x(0) \,.\, R_y(0)$
- $2\,|R_{xy}(\tau)| \leq R_x(0) + R_y(0)$

Power spectral Density :-

- P.S.D $S_x(j\omega) = \int_{-\infty}^{\infty} R_x(\tau)\, e^{-j\omega\tau}\, d\tau$

  $R_x(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_x(j\omega) e^{j\omega\tau} d\omega$
- $S_y(j\omega) = S_x(j\omega)\,|H(j\omega)|^2$
- Power = $R_x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_x(j\omega)\, d\omega$
- $R_x(\tau) = k\,\delta(\tau)$ → white process

Properties :
- $S_x(j\omega)$ even
- $S_x(j\omega) \geq 0$

Control Systems

Time Response of 2nd order system :-

Step i/P :

- $C(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} (\sin \omega_n \sqrt{1-\zeta^2}\, t \pm \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right))$
- $e(t) = \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left(\sin \omega_d t \pm \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)$
- $e_{ss} = \lim_{t\to\infty} \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left(\sin \omega_d t \pm \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)$

$\rightarrow \zeta \rightarrow$ Damping ratio ; $\zeta\omega_n \rightarrow$ Damping factor

$\zeta < 1$(Under damped) :-

$C(t) = 1- = \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \text{ Sin}\left(\omega_d t \pm \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)$

$\zeta = 0$ (un damped) :-

$c(t) = 1- \cos \omega_n t$

$\zeta = 1$ (Critically damped) :-

$C(t) = 1 - e^{-\omega_n t}(1 + \omega_n t)$

$\zeta > 1$ (over damped) :-

$$C(t) = 1 - \frac{e^{-\left(\zeta-\sqrt{\zeta^2-1}\right)\omega_n t}}{2\sqrt{\zeta^2-1}\left(\zeta-\sqrt{\zeta^2-1}\right)}$$

$$T = \frac{1}{\left(\zeta-\sqrt{\zeta^2-1}\right)\omega_n}$$

$T_{undamped} > T_{overdamped} > T_{underdamped} > T_{criticaldamp}$

Time Domain Specifications :-

- Rise time $t_r = \frac{\pi-\phi}{\omega_n\sqrt{1-\zeta^2}}$ $\phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$
- Peak time $t_p = \frac{n\pi}{\omega_d}$
- Max over shoot % $M_p = e^{-\zeta\omega_n/\sqrt{1-\zeta^2}} \times 100$
- Settling time $t_s = 3T$ 5% tolerance
  $= 4T$ 2% tolerance
- Delay time $t_d = \frac{1+0.7\zeta}{\omega_n}$
- Damping factor² $\zeta^2 = \frac{(\ln M_p)^2}{\pi^2 + (\ln M_p)^2}$
- Time period of oscillations $T = \frac{2\pi}{\omega_d}$
- No of oscillations $= \frac{t_s}{2\pi/\omega_d} = \frac{t_s \times \omega_d}{2\pi}$
- $t_r \approx 1.5\, t_d$ $t_r = 2.2\, T$
- Resonant peak $M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$; $\omega_r = \omega_n\sqrt{1-2\zeta^2}$ $\left.\begin{matrix}\omega_n > \omega_r \\ \omega_b > \omega_n\end{matrix}\right]$ $\omega_r < \omega_n < \omega_b$
- Bandwidth $\omega_b = \omega_n\,(1-2\zeta^2 + \sqrt{4\zeta^4 - 4\zeta^2 + 2})^{1/2}$

Static error coefficients :-

- Step i/p : $e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} \frac{SR(s)}{1+GH}$

  $e_{ss} = \frac{1}{1+K_p}$ (positional error) $K_p = \lim_{s\to 0} G(s)\,H(s)$

- Ramp i/p (t) : $e_{ss} = \frac{1}{K_v}$ $K_v = \lim_{s\to 0} S\,G(s)H(s)$

- Parabolic i/p $(t^2/2)$: $e_{ss} = 1/K_a$ $K_a = \lim_{s\to 0} s^2\,G(s)H(s)$

Type < i/p → $e_{ss} = \infty$
Type = i/p → e_{ss} finite
Type > i/p → $e_{ss} = 0$

- Sensitivity $S = \frac{\partial A/A}{\partial K/K}$ sensitivity of A w.r.to K.
- Sensitivity of over all T/F w.r.t forward path T/F G(s) :

Open loop: $S = 1$

Closed loop : $S = \frac{1}{1+G(s)H(s)}$

- Minimum 'S' value preferable
- Sensitivity of over all T/F w.r.t feedback T/F H(s) : $S = \frac{G(s)H(s)}{1+G(s)H(s)}$

Stability
RH Criterion :-

- Take characteristic equation 1+ G(s) H(s) = 0
- All coefficients should have same sign
- There should not be missing 's' term . Term missed means presence of at least one +ve real part root
- If char. Equation contains either only odd/even terms indicates roots have no real part & posses only imag parts there fore sustained oscillations in response.
- Row of all zeroes occur if
  (a) Equation has at least one pair of real roots with equal image but opposite sign
  (b) has one or more pair of imaginary roots
  (c) has pair of complex conjugate roots forming symmetry about origin.

Electromagnetic Fields

Vector Calculus:-
→ A. (B × C) = C. (A × B) = B. (C × A)
→ A×(B×C) = B(A.C) – C(A.B) → Bac – Cab rule
→ Scalar component of A along B is $A_B = A \cos\theta_{AB} = A \cdot a_B = \frac{(A.B)}{|B|}$
→ Vector component of A along B is $\bar{A}_B = A \cos\theta_{AB} \cdot a_B = \frac{(A.B)\,B}{|B|^2}$

Laplacian of scalars :-

- $\oint A.ds = {}_v\int(\nabla.A)dv$ → Divergence theorem
- $L\oint A.dI = {}_s\int(\nabla\times A)ds$ → Stokes theorem
- $\nabla^2 A = \nabla(\nabla.A) - \nabla\times\nabla\times A$
- $\nabla.A = 0$ → solenoidal / Divergence loss ; $\nabla.A > 0$ → source ; $\nabla.A < 0 \Rightarrow$ sink
- $\nabla\times A = 0$ → irrotational / conservative/potential.
- $\nabla^2 A = 0$ → Harmonic .

Electrostatics :-

- Force on charge 'Q' located @ r $F = \frac{Q}{4\pi\varepsilon_0}\sum_{k=1}^{N}\frac{Q_k(r-r_k)}{|r-r_k|^3}$; $F_{12} = \frac{Q_1Q_2}{4\pi\varepsilon_0 R^3} \cdot \bar{R}_{12}$
- E @ point 'r' due to charge located @ $r's$ $\bar{E} = \frac{1}{4\pi\varepsilon_0}\sum_{K=1}^{N}\frac{(r-r_k)}{|r-r_k{}^3|}Q_k$
- E due to ∞ line charge @ distance ' ρ ' $E = \frac{\rho_L}{2\pi\varepsilon_0\,\rho} \cdot a_\rho$ (depends on distance)
- E due to surface charge ρ_s is $E = \frac{\rho_s}{2\varepsilon_0}a_n$. a_n → unit normal to surface (independent of distance)
- For parallel plate capacitor @ point 'P' b/w 2 plates of 2 opposite charges is

$E = \frac{\rho_s}{2\varepsilon_0} a_n - \left(\frac{\rho_s}{2\varepsilon_0}\right)(-a_n)$

- 'E' due to volume charge $E = \frac{Q}{4\pi\varepsilon_0 R^2} a_r$.

→ Electric flux density $D = \varepsilon_0 E$ D → independent of medium

Flux $\Psi = {}_s\int D.ds$

Gauss Law :-

→ Total flux coming out of any closed surface is equal to total charge enclosed by surface .

$\Psi = Q_{enclosed} \Rightarrow \int D.ds = Q_{enclosed} = \int \rho_v . dv$

$\rho_v = \nabla . D$

→ Electric potential $V_{AB} = \frac{W}{Q} = -\int_A^B E.dI$ (independent of path)

$V_{AB} = -\int_A^B \frac{Q}{4\pi\varepsilon_0 r^2} a_r . dr\ a_r = V_B - V_A$ (for point charge)

- Potential @ any point (distance = r), where Q is located same where , whose position is vector @ r′
  $V = \frac{Q}{4\pi\varepsilon_0 |r-r'|}$

→ $V(r) = \frac{Q}{4\pi\varepsilon_0 r} + C$. [if 'C' taken as ref potential]

→ $\nabla \times E = 0$, $E = -\nabla V$

→ For monopole $E \propto \frac{1}{r^2}$; Dipole $E \propto \frac{1}{r^3}$.

$V \propto \frac{1}{r}$; $V \propto \frac{1}{r^2}$

- Electric lines of force/ flux /direction of E always normal to equipotential lines .
- Energy Density $W_E = \frac{1}{2}\sum_{k=1}^{N} Q_k V_k = \frac{1}{2}\int D.E\,dv = \frac{1}{2}\int \varepsilon_0 E^2\,dv$
- Continuity Equation $\nabla.J = -\frac{\partial \rho_v}{\partial t}$.
- $\rho_v = \rho_{v_0} e^{-t/T_r}$ where T_r = Relaxation / regeneration time = ε/σ (less for good conductor)

Boundary Conditions :- $E_{t_1} = E_{t_2}$

- Tangential component of 'E' are continuous across dielectric-dielectric Boundary .
- Tangential Components of 'D' are dis continues across Boundary .
- $E_{t_1} = E_{t_2}$; $\frac{D_{1t}}{D_{2t}} = \varepsilon_1/\varepsilon_2$.
- Normal components are of 'D' are continues , where as 'E' are dis continues.
- $D_{1n} - D_{2n} = \rho_s$; $E_{1n} = \frac{\varepsilon_2}{\varepsilon_1} E_{2n}$; $\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$
- $H_{1t} = H_{2t}$ $B_{12} = \frac{\mu_1}{\mu_2} B_2 t$

  $B_{1n} = B_{2n}$ $H_{1n} = \frac{\mu_2}{\mu_1} H_{2n}$

Maxwell's Equations :-

→ faraday law $V_{emf} = \oint E.dI = -\frac{d}{dt}\int B.ds$

→ Transformer emf $= \oint E.dI = -\int \frac{\partial B}{\partial t} ds \Rightarrow \nabla \times E = -\frac{\partial B}{\partial t}$

s

→ Motional emf = $\nabla \times E_m = \nabla \times (\mu \times B)$.

→ $\nabla \times H = J + \frac{\partial D}{\partial t}$

Electromagnetic wave propagation :-

- $\nabla \times H = J + \dot{D}$ $D = \varepsilon E$ $\nabla^2 E = \mu\varepsilon \ddot{E}$
  $\nabla \times E = -\dot{B}$ $B = \mu H$ $\nabla^2 H = \mu\varepsilon \ddot{H}$
  $\nabla . D = \rho_v$ $J = \sigma E$
  $\nabla . B = 0$
- $\frac{E_y}{H_z} = -\frac{E_z}{H_y} = \sqrt{\mu/\varepsilon}$; E.H = 0   E ⊥ H in UPW

For loss less medium $\nabla^2 E - \rho^2 E = 0$ $\rho = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha + j\beta$.

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right)}$$

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right)}$$

- $E(z, t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)$; $H_0 = E_0 / \eta$.
- $\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$ $|\eta| < \theta_\eta$
- $|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}}$ $\tan 2\theta_\eta = \sigma/\omega\varepsilon$.
- $\eta = \alpha + j\beta$ α → attenuation constant → Neper /m . $|N_p| = 20 \log_{10} e = 8.686$ dB
- For loss less medium σ = 0; α = 0.
- β → phase shift/length ; $\mu = \omega / \beta$; $\lambda = 2\pi/\beta$.
- $\frac{J_s}{J_d} = \left|\frac{\sigma E}{j\omega\epsilon E}\right| = \sigma / \omega\epsilon = \tan\theta$ → loss tanjent $\theta = 2\theta_\eta$
- If tan θ is very small (σ < < ωϵ) → good (lossless) dielectric
- If tan θ is very large (σ >> ωϵ) → good conductor
- Complex permittivity $\epsilon_C = \epsilon\left(1 - \frac{j\sigma}{\omega\epsilon}\right) = \varepsilon' - j\,\varepsilon''$.
- $\text{Tan}\,\theta = \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\epsilon}$.

Plane wave in loss less dielectric :- (σ ≈ 0)

- $\alpha = 0$; $\beta = \omega\sqrt{\mu\epsilon}$; $\omega = \frac{1}{\sqrt{\mu\epsilon}}$; $\lambda = 2\pi/\beta$; $\eta = \sqrt{\mu_r/\varepsilon_r}\ \angle 0$.
- E & H are in phase in lossless dielectric

Free space :- ($\sigma = 0$, $\mu = \mu_0$, $\varepsilon = \varepsilon_0$)

- $\alpha = 0$, $\beta = \omega\sqrt{\mu_0\varepsilon_0}$; $u = 1/\sqrt{\mu_0\varepsilon_0}$, $\lambda = 2\pi/\beta$; $\eta = \sqrt{\mu_0/\varepsilon_0} < 0 = 120\pi\angle 0$

Here also E & H in phase .

Good Conductor :-

$\sigma >> \omega\epsilon$ $\sigma/\omega\epsilon \to \infty \Rightarrow \sigma = \infty$ $\varepsilon = \varepsilon_0$; $\mu = \mu_0\mu_r$

- $\alpha = \beta = \sqrt{\pi f\mu\sigma}$; $u = \sqrt{2\omega/\mu\sigma}$; $\lambda = 2\pi / \beta$; $\eta = \sqrt{\frac{W\mu}{\sigma}}\angle 45^0$
- Skin depth $\delta = 1/\alpha$
- $\eta = \frac{1}{\sigma\delta}\sqrt{2}\, e^{j\pi/4} = \frac{1+j}{\sigma\delta}$
- Skin resistance $R_s = \frac{1}{\sigma\delta} = \sqrt{\frac{\pi f\mu}{\sigma}}$
- $R_{ac} = \frac{R_s.l}{w}$
- $R_{dc} = \frac{l}{\sigma s}$.

Poynting Vector :-

- $\int_S (E \times H)\, ds = -\frac{\partial}{dt}\int_V \frac{1}{2}[\varepsilon E^2 + \mu H^2]\, dv - \int \sigma E^2\, dv$
- $\delta_{ave}(z) = \frac{1}{2}\frac{E_0^2}{|\eta|} e^{-2\alpha z}\cos\theta_\eta\, a_z$
- Total time avge power crossing given area $P_{avge} = \int_S P_{ave}(s)\, ds$

Direction of propagation :- (a_k)

$a_k \times a_E = a_H$

$a_E \times a_H = a_k$

→ Both E & H are normal to direction of propagation

→ Means they form EM wave that has no E or H component along direction of propagation .

Reflection of plane wave :-

(a) Normal incidence

Reflection coefficient $\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$

T_{xn} coefficient $T = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2}{\eta_2 + \eta_1}$

Medium-I Dielectric , Medium-2 Conductor :-

$\eta_2 > \eta_1$:-

$\Gamma > 0$, there is a standing wave in medium & T_{xed} wave in medium '2'.

Max values of $|E_1|$ occurs

$Z_{max} = -n\pi/\beta_1 = \frac{-n\lambda_1}{2}$; n = 0, 1, 2….

$Z_{min} = \frac{-(2n+1)\pi}{2\beta_1} = \frac{-(2n+1)\lambda_1}{4}$

$\boldsymbol{\eta_2 < \eta_1}$ **:-** E_{1max} occurs @ $\beta_1 Z_{max} = \frac{-(2n+1)\pi}{2} \Rightarrow Z_{max} = \frac{-(2n+1)\pi}{2\beta_1} = \frac{-(2n+1)\lambda_1}{4}$

$$\beta_1 Z_{min} = n\pi \Rightarrow Z_{min} = \frac{-n\pi}{\beta_1} = \frac{-n\lambda_1}{2}$$

H_1 min occurs when there is $|t_1|$max

$S = \frac{|E_1|max}{|E_1|min} = \frac{|H_1|max}{|H_1|min} = \frac{1+|\Gamma|}{1-|\Gamma|}$; $|\Gamma| = \frac{s-1}{s+1}$

Since $|\Gamma| < 1 \Rightarrow 1 \le \delta \le \infty$

Transmission Lines :-

- Supports only TEM mode
- $LC = \mu\varepsilon$; $G/C = \sigma/\varepsilon$.
- $\frac{d^2V_s}{dz^2} - r^2 V_s = 0$; $\frac{d^2I_s}{dz^2} - r^2 I_s = 0$
- $\Gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$
- $V(z, t) = V_0^+ e^{-\alpha z} \cos(\omega t - \beta z) + V_0^- e^{\alpha z} \cos(\omega t + \beta z)$
- $Z_0 = -\frac{V_0^-}{I_0^-} = \frac{R+j\omega L}{\gamma} = \frac{\gamma}{G+j\omega C} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$

Lossless Line : (R = 0 =G; σ = 0)

$\rightarrow \gamma = \alpha + j\beta = j\omega\sqrt{LC}$; $\alpha = 0, \beta = w\sqrt{LC}$; $\lambda = 1/f\sqrt{LC}$, $u = 1/\sqrt{LC}$

$Z_0 = \sqrt{L/C}$

Distortion less :(R/L = G/C)

$\rightarrow \alpha = \sqrt{RG}$; $\beta = \omega L\sqrt{\frac{G}{R}} = \omega C\sqrt{\frac{R}{G}} = \omega\sqrt{LC}$

$\rightarrow Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$; $\lambda = 1/f\sqrt{LC}$; $u = \frac{1}{\sqrt{LC}} = V_p$; $uz_0 = 1/C$, $u/z_0 = 1/L$

i/p impedance :-

$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tan hl}{Z_0 + Z_L \tan hl}\right]$ for lossless line $\gamma = j\beta \Rightarrow \tan hj\beta l = j\tan\beta l$

$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + Z_L \tan\beta l}\right]$

- VSWR $= \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$
- CSWR = $-\Gamma_L$
- Transmission coefficient $S = 1 + \Gamma$
- SWR $= \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{Z_L}{Z_0}$ $(Z_L > Z_0)$ $= \frac{Z_0}{Z_L}$ $(Z_L < Z_0)$

- $|Z_{in}|_{max} = \frac{V_{max}}{I_{min}} = SZ_0$
- $|Z_{in}|_{min} = \frac{V_{min}}{I_{max}} = Z_0/S$

Shorted line :- $\Gamma_L = -1$, $S = \infty$ $Z_{in} = Z_{sc} = jZ_0 \tan\beta l$

- Γ_L = -1 , S = ∞ $Z_{in} = Z_{sc} = j\, Z_0 \tan \beta l$.
- Z_{in} may be inductive or capacitive based on length '0'

  If $l < \lambda / 4 \rightarrow$ inductive (Z_{in} +ve)
  $\frac{\lambda}{4} < l < \lambda/2 \rightarrow$ capacitive (Z_{in} -ve)

Open circuited line :-
$Z_{in} = Z_{oc} = -jZ_0 \cot \beta l$
$\Gamma_l = 1$ s = ∞ $l < \lambda / 4$ capacitive
$\frac{\lambda}{4} < l < \lambda/2$ inductive

$Z_{sc}\, Z_{oc} = Z_0^2$

Matched line : ($Z_L = Z_0$)
$Z_{in} = Z_0$ $\Gamma = 0$; s =1
No reflection . Total wave T_{xed} . So, max power transfer possible .

Behaviour of Transmission Line for Different lengths :-

$l = \lambda /4 \rightarrow \left.\begin{matrix} Z_{sc}=\infty \\ Z_{oc}=0 \end{matrix}\right\} \rightarrow$ impedance inverter @ $l = \lambda /4$

$l = \lambda /2$: $Z_{in} = Z_0 \Rightarrow \left.\begin{matrix} Z_{sc}=0 \\ Z_{oc}=\infty \end{matrix}\right\}$ impedance reflector @ $l = \lambda /2$

Wave Guides :-
TM modes : ($H_z = 0$)
$E_z = E_0 \sin\left(\frac{m\pi}{a}\right)x \ \sin\left(\frac{n\pi}{b}\right) y\ e^{-nz}$

$h^2 = k_x^2 + k_y^2$ $\therefore \gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}$ where $k = \omega\sqrt{\mu\epsilon}$
m→ no. of half cycle variation in X-direction
n→ no. of half cycle variation in Y- direction .

Cut off frequency $\omega_C = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$ $\gamma = 0;\ \alpha = 0 = \beta$

- $k^2 < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \rightarrow$ Evanscent mode ; $\gamma = \alpha$; $\beta = 0$
- $k^2 > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \rightarrow$ Propegation mode $\gamma = j\beta$ $\alpha = 0$

  $$\beta = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

- $f_c = \frac{u_p'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$ u_p' = phase velocity = $\frac{1}{\sqrt{\mu\epsilon}}$ is lossless dielectric medium

- $\lambda_c = u'/f_c = \frac{2}{\sqrt{(\frac{m}{a})^2+(\frac{n}{b})^2}}$
- $\beta = \beta' \sqrt{1-\left(\frac{f_c}{f}\right)^2}$ $\beta' = \omega/$ W β' = phase constant in dielectric medium.
- $u_p = \omega/\beta$ $\lambda = 2\pi/\beta = u_p/f$ → phase velocity & wave length in side wave guide
- $\eta_{TM} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\beta}{\omega\epsilon} = \sqrt{\frac{\mu}{\varepsilon}} \sqrt{1-\left(\frac{f_c}{f}\right)^2}$

  $\eta_{TM} = \eta' \sqrt{1-\left(\frac{f_c}{f}\right)^2}$ η' → impedance of UPW in medium

TE Modes :- ($E_z = 0$)

→ $H_z = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-nz}$

→ $\eta_{TE} = \frac{w\mu}{\beta} = \eta' / \sqrt{1-\left(\frac{f_c}{f}\right)^2}$

→ $\eta_{TE} > \eta_{TM}$

→ TE_{10} Dominant mode

Antennas :-

Hertzian Dipole :- $H_{\Phi s} = \frac{jI_0\beta\, dl}{4\pi r} \sin\theta\, e^{-j\beta\gamma}$ $E_{\theta s} = \eta H_{\Phi s}$

Half wave Dipole :-

$H_{\phi s} = \frac{jI_0 e^{-j\beta\gamma} \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi\gamma \sin\theta}$; $E_{\theta s} = \eta H_{\Phi s}$

EDC & Analog

- Energy gap $\left.\begin{matrix} E_{G/si}=1.21-3.6\times10^{-4}.T \text{ ev} \\ E_{G/Ge}=0.785-2.23\times10^{-4}.T \text{ ev}\end{matrix}\right\}$ Energy gap depending on temperature
- $E_F = E_C - KT \ln\left(\frac{N_C}{N_D}\right) = E_v + KT \ln\left(\frac{N_v}{N_A}\right)$
- No. of electrons $n = N_c\, e^{-(E_c - E_f)/RT}$ (KT in ev)
- No. of holes $p = N_v\, e^{-(E_f - E_v)/RT}$
- Mass action law $n_p = n_i^2 = N_c N_v\, e^{-EG/KT}$
- Drift velocity $v_d = \mu E$ (for si $v_d \le 10^7$ cm/sec)

- Hall voltage $v_H = \frac{B.I}{w_e}$. Hall coefficient $R_H = 1/\rho$. $\rho \rightarrow$ charge density = qN_0 = ne …
- Conductivity $\sigma = \rho\mu$; $\mu = \sigma R_H$.
- Max value of electric field @ junction $E_0 = -\frac{q}{\epsilon_{si}} N_d . n_{n0} = -\frac{q}{\epsilon_{si}} N_A . n_{p0}$.
- Charge storage @ junction $Q_+ = - Q_- = qA\, x_{n0} N_D = qA\, x_{p0} N_A$

EDC

- Diffusion current densities $J_p = - q\, D_p \frac{dp}{dx}$ $J_n = - q\, D_n \frac{dn}{dx}$
- Drift current Densities = $q(p\, \mu_p + n\mu_n)E$
- μ_p , μ_n decrease with increasing doping concentration .
- $\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p}$ = KT/q ≈ 25 mv @ 300 K
- Carrier concentration in N-type silicon $n_{n0} = N_D$; $p_{n0} = n_i^2 / N_D$
- Carrier concentration in P-type silicon $p_{p0} = N_A$; $n_{p0} = n_i^2 / N_A$
- Junction built in voltage $V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$
- Width of Depletion region $W_{dep} = x_p + x_n = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 + V_R)}$

  $* \left(\frac{2\varepsilon_{ft}}{q} = 12.93m\ for\ si\right)$
- $\frac{x_n}{x_p} = \frac{N_A}{N_D}$
- Charge stored in depletion region $q_J = \frac{q.N_A N_D}{N_A + N_D} . A . W_{dep}$
- Depletion capacitance $C_j = \frac{\varepsilon_s A}{W_{dep}}$; $C_{j0} = \frac{\varepsilon_s A}{W_{dep} / V_R = 0}$

  $C_j = C_{j0} / \left(1 + \frac{V_R}{V_0}\right)^m$

  $C_j = 2C_{j0}$ (for forward Bias)
- Forward current $I = I_p + I_n$; $I_p = Aq\, n_i^2 \frac{D_p}{L_p N_D}\left(e^{V/V_T} - 1\right)$

  $I_n = Aq\, n_i^2 \frac{D_n}{L_n N_A}\left(e^{V/V_T} - 1\right)$
- Saturation Current $I_s = Aq\, n_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)$
- Minority carrier life time $\tau_p = L_p^2 / D_p$; $\tau_n = L_n^2 / D_n$
- Minority carrier charge storage $Q_p = \tau_p I_p$, $Q_n = \tau_p I_n$

  $Q = Q_p + Q_n = \tau_T I$ τ_T = mean transist time
- Diffusion capacitance $C_d = \left(\frac{\tau_T}{\eta V_T}\right) I = \tau.g \Rightarrow C_d \propto I$.

  $\tau \rightarrow$ carrier life time , g = conductance = I / ηV_T
- $I_{02} = 2^{(T_2 - T_1)/10}\, I_{01}$
- Junction Barrier Voltage $V_j = V_B = V_r$ (open condition)

  = V_r - V (forward Bias)

  = V_r + V (Reverse Bias)
- Probability of filled states above 'E' $f(E) = \frac{1}{1+e^{(E-E_f)/KT}}$

- Drift velocity of e^- $v_d \leq 10^7$ cm/sec
- Poisson equation $\frac{d^2V}{dx^2} = \frac{-\rho_v}{\epsilon} = \frac{-nq}{\epsilon} \Rightarrow \frac{dv}{dx} = E = \frac{-nqx}{\epsilon}$

Transistor :-

- $I_E = I_{DE} + I_{nE}$
- $I_C = I_{Co} - \alpha\, I_E$ → Active region
- $I_C = -\alpha\, I_E + I_{Co}\,(1\text{-}\ e^{V_C/V_T})$

Common Emitter :-

- $I_C = (1+\beta)\, I_{Co} + \beta I_B$ $\beta = \frac{\alpha}{1-\alpha}$
- $I_{CEO} = \frac{I_{Co}}{1-\alpha}$ → Collector current when base open
- I_{CBO} → Collector current when $I_E = 0$ $I_{CBO} > I_{Co}$.
- $V_{BE,sat}$ or $V_{BC,sat}$ → - 2.5 mv /^{0} C ; $V_{CE,sat} \rightarrow \frac{V_{BE,sat}}{10}$ = - 0.25 mv /^{0}C
- Large signal Current gain $\beta = \frac{I_C - I_{CBo}}{I_B + I_{CBo}}$
- D.C current gain $\beta_{dc} = \frac{I_C}{I_B} = h_{FE}$
- $(\beta_{dc} = h_{FE}) \approx \beta$ when $I_B > I_{CBo}$
- Small signal current gain $\beta' = \left.\frac{\partial I_C}{\partial I_R}\right|_{V_{CE}} = h_{fe} = \frac{h_{FE}}{1-(I_{CBo}+I_B)\frac{\partial h_{FE}}{\partial I_C}}$
- Over drive factor = $\frac{\beta_{active}}{\beta_{forced \rightarrow under\ saturation}}$ $\because I_{C\ sat} = \beta_{forced}\, I_{B\ sat}$

Conversion formula :-

CC ↔ CE

- $h_{ic} = h_{ie}$; $h_{rc} = 1$; $h_{fc} = -(1+h_{fe})$; $h_{oc} = h_{oe}$

CB ↔ CE

- $h_{ib} = \frac{h_{ie}}{1+h_{fe}}$; $h_{ib} = \frac{h_{ie}\, h_{oe}}{1+h_{fe}} - h_{re}$; $h_{fb} = \frac{-h_{fe}}{1+h_{fe}}$; $h_{ob} = \frac{h_{oe}}{1+h_{fe}}$

CE parameters in terms of CB can be obtained by interchanging B & E .

Specifications of An amplifier :-

- $A_I = \frac{-h_f}{1+h_0 Z_L}$ $Z_i = h_i + h_r\, A_I Z_L$ $A_{vs} = \frac{A_V.Z_i}{Z_i+R_s} = \frac{A_I.Z_L}{Z_i+R_s} = \frac{A_{I_S}.Z_L}{R_s}$

  $A_V = \frac{A_I\, Z_L}{Z_i}$ $Y_0 = h_o - \frac{h_f\, h_r}{h_i + R_s}$ $A_{Is} = \frac{A_V.R_s}{Z_i+R_s} = \frac{A_{vs}.R_s}{Z_L}$

Choice of Transistor Configuration :-

- For intermediate stages CC can't be used as $A_V < 1$
- CE can be used as intermediate stage
- CC can be used as o/p stage as it has low o/p impedance
- CC/CB can be used as i/p stage because of i/p considerations.

Stability & Biasing :- (Should be as min as possible)

- For $S = \frac{\Delta I_C}{\Delta I_{Co}}\Big|_{V_{Bo,\beta}}$ $S' = \frac{\Delta I_C}{\Delta V_{BE}}\Big|_{I_{Co,\beta}}$ $S'' = \frac{\Delta I_C}{\Delta \beta}\Big|_{V_{BE,I_{Co}}}$

  $\Delta I_C = S.\ \Delta I_{Co} + S'\ \Delta V_{BE} + S''\ \Delta\beta$

- For fixed bias $S = \frac{1+\beta}{1-\beta\frac{dI_B}{dI_C}} = 1+\beta$

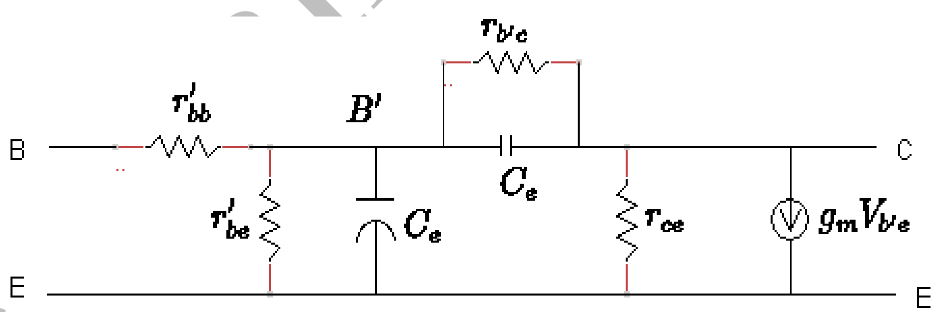
- Collector to Base bias $S = \frac{1+\beta}{1+\beta\frac{R_C}{R_C+R_B}}$ $0 < s < 1+\beta = \frac{1+\beta}{1+\beta\left(\frac{R_C+R_E}{R_C+R_E+R_B}\right)}$

- Self bias $S = \frac{1+\beta}{1+\beta\frac{R_E}{R_E+R_{th}}} \approx 1+\frac{R_{th}}{R_e}$ $\beta R_E > 10\ R_2$

- $R_1 = \frac{V_{cc}\ R_{th}}{V_{th}}$; $R_2 = \frac{V_{cc}\ R_{th}}{V_{cc}-V_{th}}$

- For thermal stability $[\ V_{cc} - 2I_c\ (R_C + R_E)]\ [\ 0.07\ I_{co}\ .\ S] < 1/\theta$; $V_{CE} < \frac{V_{CC}}{2}$

Hybrid –pi(π)- Model :-



$g_m = |I_C| / V_T$

$r_{b'e} = h_{fe} / g_m$
$r_{b'b} = h_{ie} - r_{b'e}$
$r_{b'c} = r_{b'e} / h_{re}$
$g_{ce} = h_{oe} - (1+ h_{fe})\ g_{b'c}$

For CE :-

- $f_\beta = \frac{g_{b'e}}{2\pi(C_e+C_c)} = \frac{g_m}{h_{fe}2\pi(C_e+C_c)}$
- $f_T = h_{fe}\ f_\beta$; $f_H = \frac{1}{2\pi\ r_{b'e}\ C} = \frac{g_{b'e}}{2\pi C}$ $C = C_e + C_c\ (1 + g_m\ R_L\ )$

  f_T = S.C current gain Bandwidth product
  f_H = Upper cutoff frequency

For CC :-

- $f_H = \frac{1+g_m R_L}{2\pi C_L R_L} \approx \frac{g_m}{2\pi C_L} = \frac{f_T\ C_e}{C_L} = \frac{g_m + g_{b'e}}{2\pi(C_L+C_e)}$

For CB:-

- $f_\alpha = \frac{1+h_{fe}}{2\pi r_{b'e}(C_C+C_e)} = (1+h_{fe})\ f_\beta = (1+\beta)\ f_\beta$

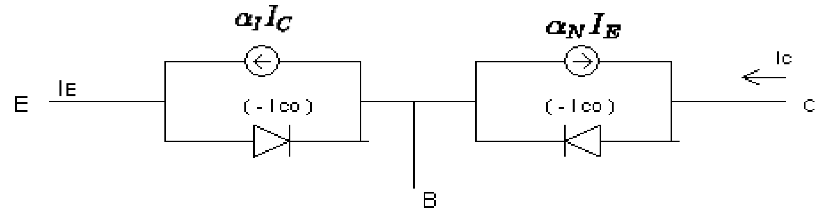
- $f_T = \frac{\beta}{1+\beta} f_\alpha$ $\qquad f_\alpha > f_T > f_\beta$

Ebress moll model :-

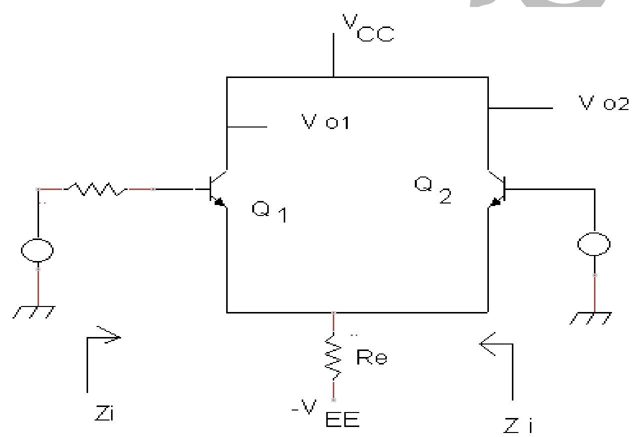
$I_C = -\alpha_N I_E + I_{Co}(1 - e^{V/V_T})$

$I_E = -\alpha_I I_C + I_{Eo}(1 - e^{V/V_T})$

$\alpha_I I_{Co} = \alpha_N I_{Eo}$



Multistage Amplifiers :-



- $f_H^* = f_H\sqrt{2^{1/n} - 1}$; $f_L^* = \frac{f_L}{\sqrt{2^{1/n}-1}}$
- Rise time $t_r = \frac{0.35}{f_H} = \frac{0.35}{B.W}$
- $t_r^* = 1.1\sqrt{t_{r1}^2 + t_{r2}^2 + \cdots}$
- $f_L^* = 1.1\sqrt{f_{L_1}^2 + f_{L_2}^2 + \cdots}$
- $\frac{1}{f_H^*} = 1.1\sqrt{\frac{1}{f_{H_1}^2} + \frac{1}{f_{H_2}^2} + \cdots}$

Differential Amplifier :-

- $Z_i = h_{ie} + (1 + h_{fe})2R_e = 2h_{fe}R_e \approx 2\beta R_e$
- $g_m = \frac{\alpha_0|I_{EE}|}{4V_T} = \frac{I_C}{4V_T} = g_m$ of BJT/4 $\qquad \alpha_0 \rightarrow$ DC value of α
- $CMRR = \frac{h_{fe}R_e}{R_s + h_{ie}}$; $R_e \uparrow, \rightarrow Z_i \uparrow, A_d \uparrow$ & $CMRR \uparrow$

Darlington Pair :-

- $A_I = (1 + \beta_1)(1 + \beta_2)$; $\qquad A_v \approx 1\ (< 1)$
- $Z_i = \frac{(1+h_{fe})^2 R_{e2}}{1 + h_{fe} h_{oc} R_{e2}}\ \Omega$ [if Q_1 & Q_2 have same type] $= A_I R_{e2}$
- $R_o = \frac{R_s}{(1+h_{fe})^2} + \frac{2h_{ie}}{1+h_{fe}}$
- $g_m = (1 + \beta_2)\, g_{m1}$

Tuned Amplifiers : (Parallel Resonant ckts used) :

- $f_0 = \frac{1}{2\pi\sqrt{LC}}$ $\qquad Q \rightarrow$ ‘Q’ factor of resonant ckt which is very high

- B.W = f_0 /Q
- $f_L = f_0 - \frac{\Delta BW}{2}$
- $f_H = f_0 + \frac{\Delta BW}{2}$
- For double tuned amplifier 2 tank circuits with same f_0 used . $f_0 = \sqrt{f_L f_H}$.

MOSFET (Enhancement) [Channel will be induced by applying voltage]

- NMOSFET formed in p-substrate
- If $V_{GS} \geq V_t$ channel will be induced & i_D (Drain → source)
- V_t → +ve for NMOS
- $i_D \propto (V_{GS} - V_t)$ for small V_{DS}
- V_{DS} ↑ → channel width @ drain reduces .

  $V_{DS} = V_{GS} - V_t$ channel width ≈ 0 → pinch off further increase no effect
- For every $V_{GS} > V_t$ there will be $V_{DS,sat}$
- $i_D = K'_n [(V_{GS} - V_t) V_{DS} - \frac{1}{2} V_{DS}^2] \left(\frac{W}{L}\right)$ → triode region ($V_{DS} < V_{GS} - V_t$)

  $K'_n = \mu_n C_{ox}$
- $i_D = \frac{1}{2} K'_n \left(\frac{W}{L}\right) [V_{DS}^2]$ → saturation
- $r_{DS} = \frac{1}{K'_n \left(\frac{W}{L}\right)(V_{GS} - V_t)}$ → Drain to source resistance in triode region

PMOS :-

- Device operates in similar manner except V_{GS} , V_{DS}, V_t are –ve
- i_D enters @ source terminal & leaves through Drain .

  $V_{GS} \leq V_t$ → induced channel $V_{DS} \geq V_{GS} - V_t$ → Continuous channel

  $i_D = K'_p \left(\frac{W}{L}\right) [(V_{GS} - V_t)^2 - \frac{1}{2} V_{DS}^2]$ $K'_p = \mu_p C_{ox}$

  $V_{DS} \leq V_{GS} - V_t$ → Pinched off channel .
- NMOS Devices can be made smaller & thus operate faster . Require low power supply .
- Saturation region → Amplifier
- For switching operation Cutoff & triode regions are used
- **NMOS** **PMOS**

$V_{GS} \geq V_t$	$V_{GS} \leq V_t$	$\rightarrow$ induced channel
$V_{GS} - V_{DS} > V_t$	$V_{GS} - V_{DS} < V_t$	$\rightarrow$ Continuous channel(Triode region)
$V_{DS} \geq V_{GS} - V_t$	$V_{DS} \leq V_{GS} - V_t$	$\rightarrow$ Pinchoff (Saturation)

Depletion Type MOSFET :- [channel is physically implanted . i_0 flows with $V_{GS} = 0$]

- For n-channel V_{GS} $\rightarrow$ +ve $\rightarrow$ enhances channel .
  $\rightarrow$ -ve $\rightarrow$ depletes channel
- $i_D - V_{DS}$ characteristics are same except that V_t is –ve for n-channel
- Value of Drain current obtained in saturation when $V_{GS} = 0 \Rightarrow I_{DSS}$.

$$\therefore I_{DSS} = \frac{1}{2} K_n' \left(\frac{W}{L}\right) V_t^2 .$$

MOSFET as Amplifier :-

- For saturation $V_D > V_{GS} - V_t$
- To reduce non linear distortion $v_{gs} << 2(V_{GS} - V_t)$
- $i_d = K_n' \left(\frac{W}{L}\right)(V_{GS} - V_t)\, v_{gs} \Rightarrow \qquad g_m = K_n' \left(\frac{W}{L}\right)(V_{GS} - V_t)$
- $\frac{v_d}{v_{gs}} = -g_m R_D$
- Unity gain frequency $f_T = \frac{g_m}{2\pi(C_{gs}+C_{gd})}$

JFET :-

- $V_{GS} \leq V_p \Rightarrow i_D = 0 \rightarrow$ Cut off
- $V_p \leq V_{GS} \leq 0, \quad V_{DS} \leq V_{GS} - V_p$

$$i_D = I_{DSS}\left[2\left(1 - \frac{V_{GS}}{V_p}\right)\left(\frac{V_{DS}}{-V_p}\right) - \left(\frac{V_{DS}}{V_p}\right)^2\right] \Bigg\} \rightarrow \text{Triode}$$

- $V_p \leq V_{GS} \leq 0 \;, \quad V_{DS} \geq V_{GS} - V_p$

$$\left.\begin{aligned} i_D = I_{DSS}\left(1-\frac{V_{GS}}{V_p}\right)^2 \Rightarrow V_{GS} = V_p\left(1-\sqrt{\frac{I_D}{I_{DSS}}}\right) \\ g_m = \frac{2I_{DSS}}{|V_p|}\left(1-\frac{V_{GS}}{V_p}\right) = \frac{2I_{DSS}}{|V_p|}\sqrt{\frac{I_D}{I_{DSS}}} \end{aligned}\right\} \rightarrow \text{Saturation}$$

Zener Regulators :-

- For satisfactory operation $\frac{V_i - V_z}{R_s} \geq I_{Z_{min}} + I_{L_{max}}$

- $R_{S_{max}} = \frac{V_{s_{min}} - V_{z_0} - I_{Z_{min}} r_z}{I_{Z_{min}} + I_{L_{max}}}$
- Load regulation = - ($r_z \parallel R_s$)
- Line Regulation = $\frac{r_z}{R_s + r_z}$.
- For finding min R_L take $V_{s\,min}$ & V_{zk} , I_{zk} (knee values (min)) calculate according to that .

Operational Amplifier:- (VCVS)

- Fabricated with VLSI by using epitaxial method
- High i/p impedance , Low o/p impedance , High gain , Bandwidth , slew rate .
- FET is having high i/p impedance compared to op-amp .
- Gain Bandwidth product is constant .
- Closed loop voltage gain $A_{CL} = \frac{A_{OL}}{1 \pm \beta A_{OL}}$ β → feed back factor
- $\Rightarrow V_0 = \frac{-1}{RC} \int V_i \, dt$ → LPF acts as integrator ;
- $\Rightarrow V_0 = \frac{-R}{L} \int V_i \, dt$; $V_0 = \frac{-L}{R} \frac{dv_i}{dt}$ (HPF)
- For Op-amp integrator $V_0 = \frac{-1}{\tau} \int V_i \, dt$; Differentiator $V_0 = -\tau \frac{dv_i}{dt}$
- Slew rate SR = $\frac{\Delta V_0}{\Delta t} = \frac{\Delta V_0}{\Delta t} \cdot \frac{\Delta V_i}{\Delta t} = A . \frac{\Delta V_i}{\Delta t}$
- Max operating frequency $f_{max} = \frac{\text{slew rate}}{2\pi \, . \, \Delta V_0} = \frac{\text{slew rate}}{2\pi \times \Delta V_i \times A}$.
- In voltage follower Voltage series feedback
- In non inverting mode voltage series feedback
- In inverting mode voltage shunt feed back
- $V_0 = -\eta V_T \ln\left(\frac{V_i}{RI_0}\right)$
- $V_0 = -V_{BE}$

  $= -\eta V_T \ln\left(\frac{V_s}{RI_{C0}}\right)$
- Error in differential % error = $\frac{1}{CMRR}\left(\frac{V_c}{V_d}\right) \times 100$ %

Power Amplifiers :-

- Fundamental power delivered to load $P_1 = \left(\frac{B_1}{\sqrt{2}}\right)^2 R_L = \frac{B_1^2}{2} R_L$
- Total Harmonic power delivered to load $P_T = \left[\frac{B_1^2}{2} + \frac{B_2^2}{2} + \cdots..\right] R_L$

$$= P_1 \left[1 + \left(\frac{B_2}{B_1}\right)^2 + \left(\frac{B_3}{B_1}\right)^2 + \ldots\ldots\right]$$
$$= [\,1 + D^2]\, P_1$$

Where $D = \sqrt{+D_2^2 + \cdots.. + D_n^2}$ $\qquad D_n = \frac{B_n}{B_1}$

D = total harmonic Distortion .

Class A operation :-

- o/p I_C flows for entire 360^0
- 'Q' point located @ centre of DC load line i.e., $V_{ce} = V_{cc} / 2$; $\eta = 25\ \%$
- Min Distortion , min noise interference , eliminates thermal run way
- Lowest power conversion efficiency & introduce power drain
- $P_T = I_C V_{CE} - i_c V_{ce}$ if $i_c = 0$, it will consume more power
- P_T is dissipated in single transistors only (single ended)

Class B:-

- I_C flows for 180^0 ; 'Q' located @ cutoff ; $\eta = 78.5\%$; eliminates power drain
- Higher Distortion , more noise interference , introduce cross over distortion
- Double ended . i.e ., 2 transistors . $I_C = 0$ [transistors are connected in that way] $P_T = i_c V_{ce}$
- $P_T = i_c V_{ce} = 0.4\ P_0$ $\qquad P_T \rightarrow$ power dissipated by 2 transistors .

Class AB operation :-

- I_C flows for more than 180^0 & less than 360^0
- 'Q' located in active region but near to cutoff ; $\eta = 60\%$
- Distortion & Noise interference less compared to class 'B' but more in compared to class 'A'
- Eliminates cross over Distortion

Class 'C' operation :-

- I_C flows for < 180 ; 'Q' located just below cutoff ; $\eta = 87.5\%$
- Very rich in Distortion ; noise interference is high .

Oscillators :-

- For RC-phase shift oscillator $f = \frac{1}{2\pi RC\sqrt{6+4K}}$ $\qquad h_{fe} \geq 4k + 23 + \frac{29}{k}$ $\qquad$ where $k = R_c/R$

$$f = \frac{1}{2\pi RC\sqrt{6}} \qquad \mu > 29$$

- For op-amp RC oscillator $f = \frac{1}{2\pi RC\sqrt{6}}$ $|A_f| \geq 29 \Rightarrow R_f \geq 29\,R_1$

Wein Bridge Oscillator :-

$f = \frac{1}{2\pi\sqrt{R'R''C'C''}}$ $h_{fe} \geq 3$

$\mu \geq 3$

$A \geq 3 \Rightarrow R_f \geq 2\,R_1$

Hartley Oscillator :-

$f = \frac{1}{2\pi\sqrt{(L_1+L_2)C}}$ $|h_{fe}| \geq \frac{L_2}{L_1}$

$|\mu| \geq \frac{L_2}{L_1}$

$|A| \geq \frac{L_2}{L_1}$

$\downarrow$

$\frac{R_f}{R_1}$

Colpits Oscillator :-

$f = \frac{1}{2\pi\sqrt{L\frac{C_1C_2}{C_1+C_2}}}$ $|h_{fe}| \geq \frac{C_1}{C_2}$

$|\mu| \geq \frac{C_1}{C_2}$

$|A| \geq \frac{C_1}{C_2}$

MatheMatics

Matrix :-

- If $|A| = 0 \rightarrow$ Singular matrix ; $|A| \neq 0$ Non singular matrix
- **Scalar Matrix** is a Diagonal matrix with all diagonal elements are equal
- **Unitary Matrix** is a scalar matrix with Diagonal element as '1' ($A^Q = (A^*)^T = A^{-1}$)
- If the product of 2 matrices are zero matrix then at least one of the matrix has det zero
- Orthogonal Matrix if $AA^T = A^T.A = I \Rightarrow A^T = A^{-1}$
- $A = A^T \rightarrow$ Symmetric
  $A = -A^T \rightarrow$ Skew symmetric

Properties :- (if A & B are symmetrical)

- A + B symmetric
- KA is symmetric
- AB + BA symmetric
- AB is symmetric iff AB = BA
- For any 'A' $\rightarrow A + A^T$ symmetric ; $A - A^T$ skew symmetric.
- Diagonal elements of skew symmetric matrix are zero
- If A skew symmetric $A^{2n} \rightarrow$ symmetric matrix ; $A^{2n-1} \rightarrow$ skew symmetric
- If 'A' is null matrix then Rank of A = 0.

Consistency of Equations :-

- $r(A, B) \neq r(A)$ is consistent
- $r(A, B) = r(A)$ consistent &
  if r(A) = no. of unknowns then unique solution
  r(A) < no. of unknowns then ∞ solutions .

Hermition , Skew Hermition , Unitary & Orthogonal Matrices :-

- $A^T = A^*$ → then Hermition
- $A^T = -A^*$ → then Hermition
- Diagonal elements of Skew Hermition Matrix must be purely imaginary or zero
- Diagonal elements of Hermition matrix always real .
- A real Hermition matrix is a symmetric matrix.
- $|KA| = K^n\,|A|$

Eigen Values & Vectors :-

- Char. Equation $|A - \lambda I| = 0$.

Roots of characteristic equation are called eigen values . Each eigen value corresponds to non zero solution X such that $(A - \lambda I)X = 0$. X is called Eigen vector .

- Sum of Eigen values is sum of Diagonal elements (trace)
- Product of Eigen values equal to Determinent of Matrix .
- Eigen values of A^T & A are same
- λ is Eigen value of A then $1/\lambda \rightarrow A^{-1}$ & $\frac{|A|}{\lambda}$ is Eigen value of adj A.
- λ_1, λ_2 …… λ_n are Eigen values of A then

$KA \rightarrow K\lambda_1, K\lambda_2 \ldots\ldots K\lambda_n$

$A^m \rightarrow \lambda_1^m, \lambda_2^m \ldots\ldots\ldots\ldots \lambda_n^m$.

$A + KI \rightarrow \lambda_1 + k,\ \lambda_2 + k, \ldots\ldots\ \lambda_n + k$

$(A - KI)^2 \rightarrow (\lambda_1 - k)^2, \ldots\ldots\ (\lambda_n - k)^2$

- Eigen values of orthogonal matrix have absolute value of ‘1’ .
- Eigen values of symmetric matrix also purely real .
- Eigen values of skew symmetric matrix are purely imaginary or zero .
- $\lambda_1, \lambda_2, \ldots\ldots \lambda_n$ distinct eigen values of A then corresponding eigen vectors $X_1, X_2, \ldots\ldots X_n$ for linearly independent set .
- adj (adj A) = $|A|^{n-2}$; | adj (adj A) | = $|A|^{(n-1)^2}$

Complex Algebra :-

- Cauchy Rieman equations

$$\left.\begin{array}{c} \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y};\ \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \\ \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta} \end{array}\right\}$$ Neccessary & Sufficient Conditions for f(z) to be analytic

- $\int_c f(z)/(Z-a)^{n+1}\,dz = \frac{2\pi i}{n!}[\,f^n(a)\,]$ if f(z) is analytic in region ‘C’ & Z =a is single point
- $f(z) = f(z_0) + f'(z_0)\frac{(z-z_0)}{1!} + f''(z_0)\frac{(z-z_0)^2}{2!} + \ldots\ldots + f^n(z_0)\frac{(z-z_0)^n}{n!} + \ldots\ldots\ldots$ Taylor Series

$\Downarrow$

if $z_0 = 0$ then it is called Mclauren Series $f(z) = \sum_0^\infty a_n (z - z_0)^n$; when $a_n = \frac{f_n(z_0)}{n\,!}$

- If f(z) analytic in closed curve ‘C’ except @ finite no. of poles then

$\int_c f(z)dz = 2\pi i$ (sum of Residues @ singular points within 'C')

Res f(a) = $\lim_{z \to a}(Z - a\, f(z)$

$= \Phi(a) / \varphi'(a)$

$= \lim_{Z \to a} \frac{1}{(n-1)!} \frac{d^{n-1}}{dz^{n-1}} ((Z-a)^n f(z))$

Calculus :-

Rolle's theorem :-

If f(x) is

(a) Continuous in [a, b]

(b) Differentiable in (a, b)

(c) f(a) = f(b) then there exists at least one value C ϵ (a, b) such that $f'(c) = 0$.

Langrange's Mean Value Theorem :-

If f(x) is continuous in [a, b] and differentiable in (a, b) then there exists atleast one value 'C' in (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$

Cauchy's Mean value theorem :-

If f(x) & g(x) are two function such that

(a) f(x) & g(x) continuous in [a, b]

(b) f(x) & g(x) differentiable in (a, b)

(c) $g'(x) \neq 0 \quad \forall x$ in (a, b)

Then there exist atleast one value C in (a, b) such that

$f'(c) / g'(c) = \frac{f(b)-f(a)}{g(b)-g(a)}$

Properties of Definite integrals :-

- a < c < b $\int_a^b f(x).dx = \int_a^c f(x).dx + \int_c^b f(x).dx$
- $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

- $\int_{-a}^{a} f(x).dx = 2\int_{0}^{a} f(x)dx$ f(x) is even

  $= 0$ f(x) is odd
- $\int_{0}^{a} f(x).dx = 2\int_{0}^{a} f(x)dx$ if $f(x) = f(2a - x)$
- $= 0$ if $f(x) = -f(2a - x)$
- $\int_{0}^{na} f(x).dx = n\int_{0}^{a} f(x)dx$ if $f(x) = f(x + a)$
- $\int_{a}^{b} f(x).dx = \int_{a}^{b} f(a+b-x).dx$
- $\int_{0}^{a} x\,f(x).dx = \frac{a}{2}\int_{0}^{a} f(x).dx$ if $f(a - x) = f(x)$
- $\int_{0}^{\pi/2} \sin^n x = \int_{0}^{\pi/2} \cos^n x = \frac{(n-1)(n-3)(n-5)\ldots\ldots.2}{n\,(n-2)(n-4)\ldots\ldots.3}$ if 'n' odd

  $= \frac{(n-1)(n-3)\ldots\ldots1}{n\,(n-2)(n-4)\ldots\ldots.2}\cdot\left(\frac{\pi}{2}\right)$ if 'n' even
- $\int_{0}^{\pi/2} \sin^m x\,.\,\cos^n x\,.\,dx = \frac{\{(m-1)(m-3)\ldots(m-5)\ldots\ldots(2 \text{ or } 1)\}\{(n-1)(n-3)\ldots\ldots(2 \text{ or } 1)\}.K}{(m+n)\,(m+n-2)(m+n-4)\ldots\ldots.2 \text{ or } 1}$

  Where $K = \pi / 2$ when both m & n are even otherwise k = 1

Maxima & Minima :-

A function f(x) has maximum @ x = a if $f'(a) = 0$ and $f''(a) < 0$

A function f(x) has minimum @ x = a if $f'(a) = 0$ and $f''(a) > 0$

Constrained Maximum or Minimum :-

To find maximum or minimum of $u = f(x, y, z)$ where x, y, z are connected by $\Phi(x, y, z) = 0$

Working Rule :-

(i) Write $F(x, y, z) = f(x, y, z) + \lambda\,\phi(x, y, z)$

(ii) Obtain $F_x = 0, F_y = 0$, $F_z = 0$

(ii) Solve above equations along with $\phi = 0$ to get stationary point .

Laplace Transform :-

- $L\left\{\frac{d^n}{dt^n} f(s)\right\} = s^n f(s) - s^{n-1} f(0) - s^{n-2} f'(0) \ldots\ldots f^{n-1}(0)$
- $L\{ t^n f(t) \} = (-1)^n \frac{d^n}{ds^n} f(s)$
- $\frac{f(t)}{t} \Leftrightarrow \int_s^\infty f(s)\ ds$
- $\int_0^t f(u)\ du \Leftrightarrow f(s) / s$.

Inverse Transforms :-

- $\frac{s}{(s^2+a^2)^2} = \frac{1}{2a}$ t sin at
- $\frac{s^2}{(s^2+a^2)^2} = \frac{1}{2a}$ [sin at + at cos at]
- $\frac{1}{(s^2+a^2)^2} = \frac{1}{2a^3}$ [sin at - at cos at]
- $\frac{s}{s^2 - a^2}$ = Cos hat
- $\frac{a}{s^2 - a^2}$ = Sin hat

Laplace Transform of periodic function : $L\{ f(t) \} = \frac{\int_0^T e^{-st} f(t)dt}{1-e^{-sT}}$

Numerical Methods :-

Bisection Method :-

(1) Take two values of x_1 & x_2 such that $f(x_1)$ is +ve & $f(x_2)$ is –ve then $x_3 = \frac{x_1+x_2}{2}$ find $f(x_3)$ if $f(x_3)$ +ve then root lies between x_3 & x_2 otherwise it lies between x_1 & x_3 .

Regular falsi method :-

Same as bisection except $x_2 = x_0 - \frac{x_1-x_0}{f(x_1)-f(x_0)} f(x_0)$

Newton Raphson Method :-

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Pi cards Method :-

$y_{n+1} = y_0 + \int_{x_0}^{x} f(x, y_n)$ $\leftarrow \frac{dy}{dx} = f(x, y)$

Taylor Series method :-

$\frac{dy}{dx} = f(x, y)$ $y = y_0 + (x- x_0)\,(y')_0 + \frac{(x- x_0)^2}{2!}\,(y)''_0 + \ldots\ldots\ldots\ldots \frac{(x- x_0)^n}{n!}\,(y)^n_0$

Euler's method :-

$y_1 = y_0 + h\, f(x_0, y_0)$ $\leftarrow \frac{dy}{dx} = f(x, y$

$y_1^{(1)} = y_0 + \frac{h}{2}\,[f(x_0, y_0) + f(x_0 + h, y_1)$

$y_1^{(2)} = y_0 + \frac{h}{2}\,[f(x_0, y_0) + f(x_{0+h}, y_1^{(1)})\,]$

:

:

Calculate till two consecutive value of 'y' agree

$y_2 = y_1 + h\, f(x_0 + h, y_1)$

$y_2^{(1)} = y_0 + \frac{h}{2}\,[f(x_0 + h, y_1) + f(x_0 + 2h, y_2)$

………………

Runge's Method :-

$k_1 = h\, f(x_0, y_0)$

$k_2 = h\, f(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$

$k' = h\, f(x_0+h\,, y_0 + k_1)$

$k_3 = h\,(\, f\,(x_0+h\,, y_0 + k'\,))$

finally compute $K = \frac{1}{6}\,(K_1 + 4K_2 + K_3)$

Runge Kutta Method :-

$k_1 = h\, f(x_0, y_0)$

$k_2 = h\, f(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$

finally compute $K = \frac{1}{6}\,(K_1 + 2K_2 + 2K_3 + K_4)$

$k_3 = h\,f(x_0 + \frac{h}{2},\ y_0 + \frac{k_2}{2})$ ∴ approximation vale $y_1 = y_0 + K$.

$k_3 = h\,f(x_0 + h,\ y_0 + k_3)$

Trapezoidal Rule :-

$\int_{x_0}^{x_0+nh} f(x).\,dx = \frac{h}{2}[\,(y_0 + y_n) + 2\,(y_1 + y_2 + \ldots\ldots\, y_{n-1})]$

f(x) takes values y_0, y_1 …..

@ x_0, x_1, x_2 ……..

Simpson's one third rule :-

$\int_{x_0}^{x_0+nh} f(x).\,dx = \frac{h}{3}[\,(y_0 + y_n) + 4\,(y_1 + y_3 + \ldots\ldots\, y_{n-1}) + 2\,(y_2 + y_4 + \cdots\ldots + y_{n-2})]$

Simpson three eighth rule :-

$\int_{x_0}^{x_0+nh} f(x).\,dx = \frac{3h}{8}[\,(y_0 + y_n) + 3\,(y_1 + y_2 + y_4 + y_5 + \ldots\ldots\, y_{n-1}) + 2\,(y_3 + y_6 + \cdots\ldots + y_{n-3})\,]$

Differential Equations :-

Variable & Seperable :-

General form is $f(y)\,dy = \phi(x)\,dx$

Sol: $\int f(y)\,dy = \int \phi(x)\,dx + C$.

Homo generous equations :-

General form $\frac{dy}{dx} = \frac{f(x,y)}{\phi(x,y)}$ $f(x, y)$ & $\phi(x, y)$ Homogenous of same degree

Sol : Put $y = Vx$ $\Rightarrow \frac{dy}{dx} = V + x\frac{dv}{dx}$ & solve

Reducible to Homogeneous :-

General form $\frac{dy}{dx} = \frac{ax+by+c}{a'x+b'y+c'}$

(i) $\frac{a}{a'} \neq \frac{b}{b'}$

Sol : Put $x = X + h$ $y = Y + k$

$\Rightarrow \frac{dy}{dx} = \frac{ax+by+(ah+bk+c)}{a'x+b'y+(a'h+b'k+c')}$ Choose h, k such that $\frac{dy}{dx}$ becomes homogenous then solve by Y = VX

(ii) $\frac{a}{a'} = \frac{b}{b'}$

Sol : Let $\frac{a}{a'} = \frac{b}{b'} = \frac{1}{m}$

$$\frac{dy}{dx} = \frac{ax+by+c}{m(ax+by)+c}$$

Put ax + by = t $\Rightarrow \frac{dy}{dx} = \left(\frac{dt}{dx} - a\right)/b$

Then by variable & seperable solve the equation .

Libnetz Linear equation :-

General form $\frac{dy}{dx}$ +py = Q where P & Q are functions of "x"

I.F = $e^{\int p.dx}$

Sol : $y(I.F) = \int Q.(I.F)\,dx + C$.

Exact Differential Equations :-

General form M dx + N dy = 0 M → f (x, y)

N → f(x, y)

If $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ then

Sol : $\int M.dx$ + $\int$(terms of N containing x) dy = C

(y constant)

Rules for finding Particular Integral :-

$$\frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}$$

$$= x \frac{1}{f'(a)} e^{ax} \quad \text{if } f(a) = 0$$

$$= x^2 \frac{1}{f''(a)} e^{ax} \quad \text{if } f'(a) = 0$$

$\frac{1}{f(b^2)}\sin(ax+b) = \frac{1}{f(-a^2)}\sin(ax+b)$ $\quad f(-a^2) \neq 0$

$= x\frac{1}{f'(-a^2)}\sin(ax+b)$ $\quad f(-a^2) = 0$ $\quad$ Same applicable for cos (ax + b)

$= x^2 \frac{1}{f''(-a^2)}\sin(ax+b)$

$\frac{1}{f(D)}x^m = [f(D)]^y x^m$

$\frac{1}{f(D)}e^{ax}f(x) = e^{ax}\frac{1}{f(D+a)}f(x)$

Vector Calculus :-

Green's Theorem :-

$$\int_C (\phi\, dx + \varphi\, dy) = \int\int\left(\frac{\partial\Psi}{\partial x} - \frac{\partial\phi}{\partial y}\right)dx\, dy$$

This theorem converts a line integral around a closed curve into Double integral which is special case of Stokes theorem .

Series expansion :-

Taylor Series :-

$$f(x) = f(a) + \frac{f'(a)}{1!}(x\text{-}a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots\ldots\ldots\ldots + \frac{f^n(a)}{n!}(x-a)^n$$

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \ldots\ldots\ldots\ldots + \frac{f^n(0)}{n!}x^n + \ldots\ldots\text{ (mc lower series)}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \ldots\ldots \quad |nx| < 1$$

$$e^x = 1 + x + \frac{x^2}{2!} + \ldots\ldots$$

$$\text{Sin } x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\ldots$$

$$\text{Cos } x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\ldots$$

Digital Electronics

- Fan out of a logic gate = $\frac{I_{OH}}{I_{IH}}$ or $\frac{I_{OL}}{I_{IL}}$
- Noise margin : $V_{OH} - V_{IH}$ or $V_{OL} - V_{IL}$
- Power Dissipation $P_D = V_{cc}\, I_{cc} = V_{cc}\left[\frac{I_{CCL}+I_{CCH}}{2}\right]$ $\quad I_{CCL} \rightarrow I_c$ when o/p low

I_{CCH} → I_c when o/p high .

- TTL , ECL & CMOS are used for MSI or SSI
- Logic swing : V_{OH} - V_{OL}
- RTL , DTL , TTL → saturated logic ECL → Un saturated logic
- Advantages of Active pullup ; increased speed of operation , less power consumption .
- For TTL floating i/p considered as logic "1" & for ECL it is logic "0" .
- "MOS" mainly used for LSI & VLSI . fan out is too high
- ECL is fastest gate & consumes more power .
- CMOS is slowest gate & less power consumption
- NMOS is faster than CMOS .
- Gates with open collector o/p can be used for wired AND operation (TTL)
- Gates with open emitter o/p can be used for wired OR operation (ECL)
- ROM is nothing but combination of encoder & decoder . This is non volatile memory .
- SRAM : stores binary information interms of voltage uses FF.
- DRAM : infor stored in terms of charge on capacitor . Used Transistors & Capacitors .
- SRAM consumes more power & faster than DRAM .
- CCD , RAM are volatile memories .
- 1024 × 8 memory can be obtained by using 1024 × 2 memories
- No. of memory ICs of capacity 1k × 4 required to construct memory of capacity 8k × 8 are "16"

DAC

- FSV = $V_R\left(1-\frac{1}{2^n}\right)$
- Resolution = $\frac{\text{step size}}{\text{FSV}} = \frac{V_R/2^n}{V_R\left(1-\frac{1}{2^n}\right)} = \frac{1}{2^n-1} \times 100\%$
- Accuracy = $\pm\frac{1}{2}$ LSB = $\pm\frac{1}{2^{n+1}}$
- Analog o/p = K. digital o/p

ADC

* LSB = Voltage range / 2^n

* Resolution = $\frac{FSV}{2^n-1}$

* Quantisation error = $\frac{V_R}{2^n}$ %

PROM , PLA & PAL :-

AND	OR	
Fixed	Programmable	PROM
Programmable	fixed	PAL
Programmable	Programmable	PLA

- Flash Type ADC : 2^{n-1} → comparators
  2^n → resistors
  $2^n \times n$ → Encoder

Fastest ADC :-

- Successive approximation ADC : n clk pulses
- Counter type ADC : 2^n - 1 clk pulses

- Dual slope integrating type : 2^{n+1} clock pulses .

Flip Flops :-

- a(n+1) = S + R′ Q
  = D
  = JQ′ + K′Q
  = TQ′ + T′ Q

Excitation tables :-

		S	R
0	0	0	x
0	1	1	0
1	0	0	1
1	1	x	0

		J	K
0	0	0	x
0	1	1	x
1	0	x	1
1	1	x	0

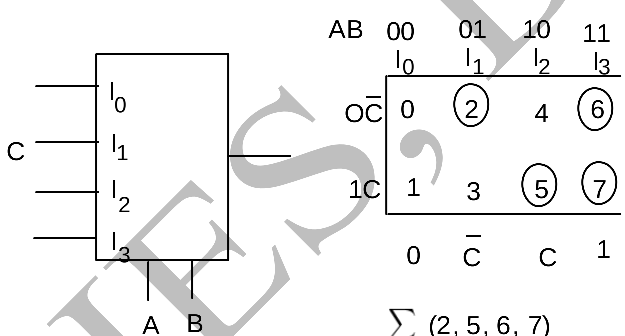
		D
0	0	0
0	1	1
1	0	0
1	1	1

		T
0	0	0
0	1	1
1	0	1
1	1	0

- For ring counter total no.of states = n
- For twisted Ring counter = "2n" (Johnson counter / switch tail Ring counter) .
- To eliminate race around condition $t_{pd\ clock} << t_{pd\ FF}$.
- In Master slave master is level triggered & slave is edge triggered

Combinational Circuits :-

Multiplexer :-



$\sum$ (2, 5, 6, 7)

- 2^n i/ps ; 1 o/p & 'n' select lines.
- It can be used to implement Boolean function by selecting select lines as Boolean variables
- For implementing 'n' variable Boolean function $2^n \times 1$ MUX is enough .
- For implementing "n + 1" variable Boolean $2^n \times 1$ MUX + NOT gate is required .
- For implementing "n + 2" variable Boolean function $2^n \times 1$ MUX + Combinational Ckt is required
- If you want to design $2^m \times 1$ MUX using $2^n \times 1$ MUX . You need 2^{m-n} $2^n \times 1$ MUXes

Decoder :-

- n i/p & 2^n o/p's
- used to implement the Boolean function . It will generate required min terms @ o/p & those terms should be "OR" ed to get the result .
- Suppose it consists of more min terms then connect the max terms to NOR gate then it will give the same o/p with less no. of gates .
- If you want to Design $m \times 2^m$ Decoder using $n \times 2^n$ Decoder . Then no. of $n \times 2^n$ Decoder required $= \frac{2^m}{2^n}$.
- In Parallel ("n" bit) total time delay = $2_n\ t_{pd}$.
- For carry look ahead adder delay = 2 t_{pd} .

Microprocessors

- Clock frequency $= \frac{1}{2}$ crystal frequency
- Hardware interrupts

TRAP (RST 4.5)	0024H	both edge level
RST 7.5	→ Edge triggered	003CH
RST 6.5	0034 H	level triggered 002C
RST 5.5		level triggered 002C
INTR	Non vectored	

- Software interrupts

RST 0	0000H	Vectored
RST 1	0008H	
2	0010H	
:	0018H	
:		
7	0038H	

-

S_1	S_0	
0	0	Halt
0	1	write
1	0	Read
1	1	fetch

- HOLD & HLDA used for Direct Memory Access . Which has highest priority over all interrupts .

Flag Registers :-

S	Z	X	AC	X	P	X	CY

- **Sign flag :-** After arthematic operation MSB is resolved for sign flag . S = 1 → -ve result
- If Z = 1 ⇒ Result = 0
- AC : Carry from one stage to other stage is there then AC = 1
- P : P =1 ⇒ even no. of one's in result .
- CY : if arthematic operation Results in carry then CY = 1
- For INX & DCX no flags effected
- In memory mapped I/O ; I/O Devices are treated as memory locations . You can connect max of 65536 devices in this technique .
- In I/O mapped I/O , I/O devices are identified by separate 8-bit address . same address can be used to identify i/p & o/p device .
- Max of 256 i/p & 256 o/p devices can be connected .

Programmable Interfacing Devices :-

- 8155 → programmable peripheral Interface with 256 bytes RAM & 16-bit counter
- 8255 → Programmable Interface adaptor
- 8253 → Programmable Interval timer
- 8251 → programmable Communication interfacing Device (USART)
- 8257 → Programmable DMA controller (4 channel)
- 8259 → Programmable Interrupt controller
- 8272 → Programmable floppy Disk controller
- CRT controller
- Key board & Display interfacing Device

RLC :- Each bit shifted to adjacent left position . D_7 becomes D_0 .

CY flag modified according to D_7

RAL :- Each bit shifted to adjacent left position . D_7 becomes CY & CY becomes D_0 .

ROC :-CY flag modified according D_0

RAR :- D_0 becomes CY & CY becomes D_7

CALL & RET Vs PUSH & POP :-

CALL & RET

- When CALL executes , µp automatically stores 16 bit address of instruction next to CALL on the Stack
- CALL executed , SP decremented by 2
- RET transfers contents of top 2 of SP to PC
- RET executes "SP" incremented by 2

PUSH & POP

* Programmer use PUSH to save the contents rp on stack

* PUSH executes "SP" decremented by "2" .

* same here but to specific "rp" .

* same here

Some Instruction Set information :-

CALL Instruction

CALL → 18T states SRRWW

CC → Call on carry 9 – 18 states

CM → Call on minus 9-18

CNC → Call on no carry

CZ → Call on Zero ; CNZ call on non zero

CP → Call on +ve

CPE → Call on even parity

CPO → Call on odd parity

RET : - 10 T

RC : - 6/ 12 'T' states

Jump Instructions :-

JMP → 10 T

JC → Jump on Carry 7/10 T states

JNC → Jump on no carry

JZ → Jump on zero

JNZ → Jump on non zero

JP → Jump on Positive

JM → Jump on Minus

JPE → Jump on even parity

JPO → Jump on odd parity .

- PCHL : Move HL to PC 6T
- PUSH : 12 T ; POP : 10 T
- SHLD : address : store HL directly to address 16 T
- SPHL : Move HL to SP 6T
- STAX : R_p store A in memory 7T
- STC : set carry 4T
- XCHG : exchange DE with HL "4T"

XTHL :- Exchange stack with HL 16 T

- For "AND " operation "AY" flag will be set & "CY" Reset
- For "CMP" if A < Reg/mem : CY → 1 & Z → 0 (Nothing but A-B)
  A > Reg/mem : CY → 0 & Z → 0

  A = Reg/mem : Z → 1 & CY → 0 .

- "DAD" Add HL + RP (10T) → fetching , busidle , busidle
- DCX , INX won't effect any flags . (6T)

- DCR, INR effects all flags except carry flag . “Cy” wont be modified
- “LHLD” load “HL” pair directly
- “ RST “ → 12T states
- SPHL , RZ, RNZ …., PUSH, PCHL, INX , DCX, CALL → fetching has 6T states
- PUSH – 12 T ; POP – 10T